ATTITUDE DETERMINATION FOR FAME
ROUGH DRAFT

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1 The Need for Attitude Determination

The assignment of time stamps to image centroids is only the first step in deriving a stellar position and parallax from FAME data. These times must then be used to obtain one-dimensional positions, or *abscissae*, along a great-circle scan. This requires accurate knowledge of the spacecraft attitude as a function of time. The abscissae form an internally rigid system with an undetermined zero point. In the final *sphere reconstruction* phase of the data analysis, all of the great-circle scans are interconnected to form a global rigid system.

The along-the-scan attitude, or *roll* angle, is clearly the most critical component of the attitude, since errors in the roll angle have a first order effect on the stellar abscissae. The abscissae must also be corrected for the field rotation, which arises from the pitch and yaw of the spacecraft. This correction is the height above field center, times the cosine of the field-rotation angle, and therefore is second-order in nature.

The method of obtaining the time-dependent attitude from the stellar data is the subject of this note. The accuracy of this solution is assessed through a software simulation. In particular, effect on attitude determination of the use of attitutude-control thrusters is addressed.

2 The Purpose of the Second FOV

Choosing an arbitrary star as the provisional zero point of a great circle scan, the instantaneous roll angle can be determined from the data by measuring the relative angle between the first and second star, then the second and third, and so on until the first star is seen again after a complete rotation. Adding up these relative angles gives the along-the-scan angle of any star relative to the first. This method is not satisfactory, however, because there is an accumulating measurement error as one progresses around the great circle. It is true that when the first star is seen for the second time, one can impose the constraint that the sum of all relative angles to that point is equal to 360°. The accumulated error can then be uniformly distributed around the great circle, but higher order deviations from the true attitude will still be present. The excursion from the true roll angle will be largest for the point 180° from the zero point.

The HIPPARCOS consortium solved this problem by introducing a second field of view separated from the first by a very stable angle known as the *basic* angle. One may imagine performing a harmonic decomposition of the attitude error as a function of angle along the scan. Each time a star seen in the first FOV is subsequently seen in the second, it may be concluded that the spacecraft has rotated through an angle equal to the basic angle. These constraints give rise to a set of normal equations which may be solved for the Fourier coefficients. This procedure has been referred to as the *great circle reduction* and has the effect of transferring errors from the spatial domain to the spatial-frequency domain. The attitude errors are therefore uniformly distributed in angle, and do not *bunch up* at the anti node of the great-circle scan. If done correctly, the variance of the one-dimensional abscissa of a star, which includes the uncertainty in attitude at that point, is comparable to the variance of the relative separation between two nearest-neighbor stars. The ratio of these variances is known as the *non-rigidity factor*, V, and is always greater than or equal to one.

The principle difference between FAME and HIPPARCOS is FAME's lack of an instantaneous FOV within which the relative angles between stars can be measured. Thus, a slightly different approach suggests itself, where deviations in the angular velocity are computed, rather than direct corrections to the attitude. The instantaneous roll angle may then be computed by integration of the derived angular velocity function. This difference in implementation is unlikely to significantly modify the estimated mission accuracy. Thus, Germain (1995) used the HIPPARCOS model to estimate the accuracy of FAME, defining an *effective field of view* (EFOV) for FAME in a mathematically precise way. This permitted the final catalog accuracy to be estimated in a manner similar to the that used for HIPPARCOS, requiring only the assumed centroiding precision as input.

3 Determination of Along-the-Scan Attitude

In the current incarnation of FAME, the focal plane is populated with two banks of CCDs. The angle between these CCD banks effectively gives us a second basic angle. This may be used to form additional constraints, improving the rigidity of the great-circle reduction. Let γ denote the large, $\sim 65°$ angle defined by the compound mirror assembly, and let θ represent the angle between the outputs of the two banks of CCDs in the focal plane. The non-rigidity factor resulting from different combinations of γ and θ was studied by Makarov, Høg, and Lindegren (1996).

According to Mook (1998, private communication), perturbations in the angular velocity are expected to occur at harmonics of the nominal rotational frequency, and have amplitudes not larger that 6 mas/s. Therefore, the angular velocity may be expanded in a Fourier series as follows:

$$\omega(t) = \omega_\circ + \sum_{n=1}^{N} \left[a_n \cos(\frac{n\omega_\circ}{2} t) + b_n \sin(\frac{n\omega_\circ}{2} t) \right] \tag{1}$$

where $\omega_\circ$ is the average rotational frequency.

The following times are measured for any star which is observed on both CCDs and in both FOVs:

time	FOV	CCD bank
$t_\circ$	1	1
t_1	1	2
t_2	2	1
t_3	2	2

Using these observations, the coefficients in equation (1) can be determined from the following normal equations:

$$\begin{aligned}
&\int_{t_\circ}^{t_1} \omega(t)\,dt = \theta \qquad (a) &&\int_{t_\circ}^{t_2} \omega(t)\,dt = \gamma \qquad (b)\\
&\int_{t_\circ}^{t_3} \omega(t)\,dt = \gamma + \theta \qquad (c) &&\int_{t_1}^{t_2} \omega(t)\,dt = \gamma - \theta \qquad (d)\\
&\int_{t_1}^{t_3} \omega(t)\,dt = \gamma \qquad (e) &&\int_{t_2}^{t_3} \omega(t)\,dt = \theta \qquad (f)
\end{aligned} \tag{2}$$

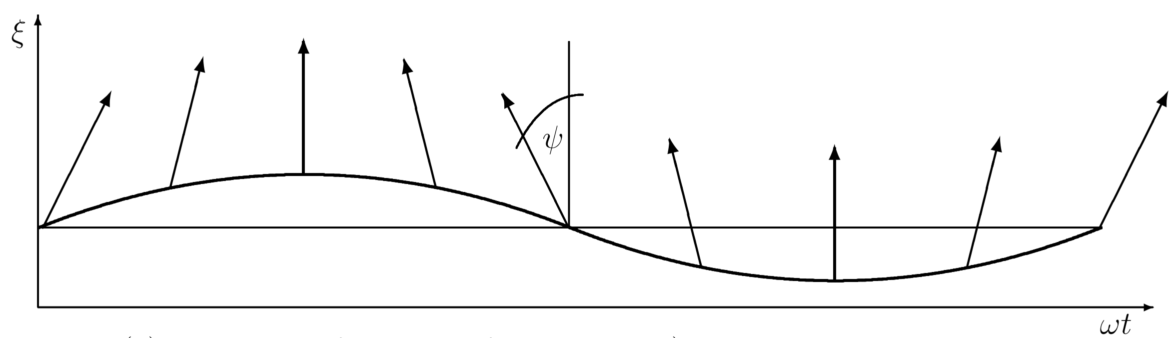
A numerical simulation of this procedure has been conducted using only equations (2a) and (2b), with $\gamma = 65^\circ$ and $\theta = 1.6^\circ$. Normally-distributed errors were added to the transit times of 1,000 uniformly-spaced stars spanning a single great-circle scan. A rotation model was assumed which consisted of periodic angular accelerations at several harmonics of a nominal 20 minute rotational period. This software provides a convenient tabular format for changing the input rotation model, and a flag may be set to introduce random variations in frequency, amplitude and phase of the rotation perturbations. For this test, results of ten such random variations of the input model were averaged.

The input rotation models were recovered in the simulation with the non-rigidity factors shown in the table below. The first column shows the length of the scan that is uninterrupted by thruster firings. The non-rigidity factor is a measure of the accuracy of attitude determination, and depends upon the number of stars used in the great circle reduction, and on the size of the basic angles. It is defined as a ratio of variances. Thus, for a 425° scan, attitude uncertainty contributes only 7.2% to the single-observation error. Note that the theoretical limit of the non-rigidity factor is unity.

scan length	non-rigidity (V)
425°	1.15
335°	1.28
245°	1.99
155°	2.13

4 Other Attitude Angles

The pitch and yaw angles are needed to compute a second-order correction to the abscissae. Let ψ be defined as the yaw, or rotation about the line of sight of the first bank of CCDs in the first FOV. This leads directly to a rotation of the field seen by this bank of detectors. The pitch, ξ, is defined as rotation about an axis perpendicular to this line of sight and in the plane defined by the two look directions. This produces a first-order change in the height at which a stellar image transits the FOV. The effect of slow variations in these angles is illustrated in the figure below. The arrows represent the orientation of the detector array. The horizontal line represents the mean value of ξ, and defines a *reference great circle* to which all abscissae are referred. Both ξ and ψ are expected to be on the order of a few arcminutes.



Pitch (ξ) and yaw as functions of roll angle ωt).

The strategy for solving for ξ and ψ from the stellar data will be to assume that these angle vary smoothly enough that they can be expressed as series expansions. The coefficients will be obtained from a solution of the normal equations developed below. Note that we can define a different pair of angles, ξ' and ψ', with respect to the second bank of detectors, or the second FOV. Let ϕ represent the angular separation of this second look direction from the first detector bank in the first FOV. In practice, ϕ will be some combination of γ and θ. It can be shown that the following identities obtain:

$$\begin{aligned} \sin(\xi') &= \sin(\xi)\cos(\psi)\cos(\phi) - \sin(\psi)\sin(\phi) \\ \sin(\psi') &= \sin(\xi)\cos(\psi)\sin(\phi) + \sin(\psi)\cos(\phi) \end{aligned} \tag{3}$$

or to first order,

$$\begin{aligned} \xi' &= \cos(\phi)\xi - \sin(\phi)\psi \\ \psi' &= \sin(\phi)\xi + \cos(\phi)\psi \end{aligned} \tag{4}$$

The *height* (ie. cross-scan position) at which an image transits a given bank of CCDs in a given FOV contains information about the angles ξ' and ψ' which can be related to ξ and ψ through equations (4). As the spacecraft rotates, and a star is seen by both CCD banks and through both FOVs at the times $t_1 - t_4$, the following heights, $h(t)$, are observed:

Measured height	FOV	CCD bank
$h_1(t_1)$	1	1
$h_2(t_2)$	1	2
$h_3(t_3)$	2	1
$h_4(t_4)$	2	2

The following normal equations are then obtained from the four observations of each star:

$$\begin{aligned} h_2(t_2) - h_1(t_1) &= \xi(t_1) - \cos(\theta)\xi(t_2) + \psi(t_2)\sin(\theta) \\ h_3(t_3) - h_1(t_1) &= \xi(t_1) - \cos(\gamma)\xi(t_3) + \xi(t_3)\sin(\gamma) \\ h_4(t_4) - h_1(t_1) &= \xi(t_1) - \cos(\gamma+\theta)\xi(t_4) + \psi(t_4)\sin(\gamma+\theta) \\ h_3(t_3) - h_2(t_2) &= \cos(\theta)\xi(t_2) - \sin(\theta)\psi(t_2) - \cos(\gamma)\xi(t_3) + \sin(\gamma)\psi(t_3) \\ h_4(t_4) - h_2(t_2) &= \cos(\theta)\xi(t_2) - \sin(\theta)\psi(t_2) - \cos(\gamma+\theta)\xi(t_4) + \sin(\gamma+\theta)\psi(t_4) \\ h_4(t_4) - h_3(t_3) &= \cos(\gamma)\xi(t_3) - \sin(\gamma)\psi(t_3) - cos(\gamma+\theta)\xi(t_4) + \sin(\gamma+\theta)\psi(t_4) \end{aligned} \tag{5}$$

Having solved for ξ and ψ as functions of time, one can use the exact equations (3) to solve for ξ' and ψ' for each observation. These are then used to project the observed image centroid onto the reference great circle.